**Practical 4**

**Fourier series decomposition and spectral**

Using a famous "AirPassengers" dataset, which contains the monthly total number of airline passengers from 1949 to 1960. We'll perform Fourier series decomposition and spectral analysis on this dataset to identify any underlying periodic components.

### **Explanation of the Code**

1. **Loading Libraries:**
   * We load the necessary libraries: ggplot2, forecast, and tseries. If the packages are not already installed, the code will install them first.
2. **Loading "AirPassengers" Dataset:**
   * We load the "AirPassengers" dataset, which comes pre-installed with R. This dataset contains the monthly total number of airline passengers from 1949 to 1960.
3. **Plotting Original Data:**
   * We plot the original "AirPassengers" data to visualize the monthly passenger counts over time.
4. **Fourier Series Decomposition:**
   * We decompose the time series using Fourier terms. The fourier function from the forecast package generates Fourier terms with a specified number of harmonics (K). Here, we use K=5 for five harmonics.
   * We fit a linear model with the Fourier terms as predictors and display the summary of the model.
5. **Spectral Analysis:**
   * We perform spectral analysis using the spectrum function from the tseries package. The periodogram is plotted to identify the dominant frequencies in the "AirPassengers" data.

### **Interpreting the Results**

* **Fourier Series Decomposition:**
  + The linear model summary will show the significance of the Fourier terms. Significant terms indicate the presence of periodic components at specific frequencies.
* **Spectral Analysis:**
  + The periodogram plot will display peaks at frequencies corresponding to periodic components in the "AirPassengers" data. Peaks at certain frequencies may indicate seasonal patterns or other periodicities.

**Example 2**

**For this example, let's consider a real-world dataset: the monthly average CO2 concentration at the Mauna Loa Observatory.**

### **Explanation of the Code**

1. **Loading Libraries:**
   * We load the necessary libraries: forecast, ggplot2, and tseries. If the packages are not already installed, the code will install them first.
2. **Downloading and Preprocessing Data:**
   * We download the Mauna Loa CO2 dataset from NOAA. The data is in a text file, and we skip the header and fill missing values appropriately.
   * We rename the columns for clarity and remove rows with missing values in the average column.
3. **Creating Time Series Object:**
   * We create a time series object co2\_ts using the ts function, specifying the start year and month, and setting the frequency to 12 (monthly data).
4. **Plotting Data:**
   * We use autoplot from the forecast package to visualize the CO2 concentration over time.
5. **Fourier Series Decomposition:**
   * We decompose the time series using Fourier terms. The fourier function generates Fourier terms with a specified number of harmonics (K). Here, we use K=2 for two harmonics.
   * We fit a linear model with the Fourier terms as predictors and display the summary of the model.
6. **Spectral Analysis:**
   * We perform spectral analysis using the spectrum function from the tseries package. The periodogram is plotted to identify the dominant frequencies in the CO2 data.

### **Interpreting the Results**

* **Fourier Series Decomposition:**
  + The linear model summary will show the significance of the Fourier terms. Significant terms indicate the presence of periodic components at specific frequencies.
* **Spectral Analysis:**
  + The periodogram plot will display peaks at frequencies corresponding to periodic components in the data. Peaks indicate strong periodic signals, such as seasonal variations.